2021

MATHEMATICS

Paper - MATH 301

(Partial Differential Equations)

(CBCS)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Answer any ***four*** questions of the following.

1. (a) State the interior Neumann's problem and obtain a necessary condition for the existence of its solution.

   (b) Prove that if the Dirichlet's problem for a bounded region has a solution, then it is unique. [5+5]

2. (a) Prove that if the Neumann problem for a bounded region has a solution, then it is either unique or it differs from one another by a constant only.

   (b) Show that if a harmonic function vanishes everywhere on the boundary, then it is identically zero everywhere. [5+5]

3. The ends A and B of a rod, 10 cm in length, are kept at temperatures 0^oC and 100^oC until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20^oC, and the end B is decreased to 60^oC. Find the temperature distribution in the rod at time t. [10]

4. (a) Obtain the solution of the interior Dirichlet problem for a sphere using the Green's function method and hence derive the Poisson integral formula.

   (b) Solve the equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad x > 0, \ t > 0.$$

   with the initial and boundary conditions

$$u(x,0) = 0, \quad x > 0$$
$$u(0,t) = T_0 \, (= \text{constant}), \ u(x,t) \to 0 \ \text{ as } \ x \to \infty, \quad t > 0.$$

[5+5]

5. A thin rectangular homogeneous thermally conducting plate lies in the xy-plane defined by $0 \leq x \leq a$, $0 \leq y \leq b$. The edge $y = 0$ is held at the temperature $\lambda\, x(x-a)$, where λ is a constant, while the remaining edges are held at 0^o. The other faces are insulated and no internal sources and sinks are present. Find the steady state temperature inside the plate. [10]

6. (a) An infinitely long string having one end at $x = 0$ is initially at rest on the x-axis. The end $x = 0$ undergoes a periodic transverse displacement described by $a_0 \sin \omega\, t$, $t > 0$. Find the displacement of any point on the string at any time t.

   (b) Using Laplace transform technique, solve the IBVP described by

$$u_{tt} = u_{xx}, \quad 0 \leq x \leq 1, \ t > 0$$

   satisfying the following conditions

$$u(0,t) = 0 = u(1,t) \, (t > 0); \quad u(x,0) = \sin \pi\, x, \ u_t(x,0) = -\sin \pi\, x \, (0 \leq x \leq 1).$$

[5+5]