# UNIVERSITY OF GOUR BANGA
## P.O.- Mokdumpur, Malda 732103, West Bengal

## Syllabus for Research Eligibility Test (RET) in Mathematics- 2021

### Detailed Syllabus for RET in Mathematics

**GROUP A:RESEARCH METHODOLOGY:**

**Introduction to Research Methodology**: Meaning of Research, Objectives of Research, Motivations in Research, Ethics of Research (Legal issues, copyright, plagiarism), Types of Research, Research Approaches, Significance of Research, Research methods v/s Methodology, Research and Scientific Methods, Research Process, Criteria of good Research, Plagiarism of research paper.
Defining the Research Problem: What is Research Problem? Selecting the Problem, Necessity of Technique in defining the problem.

**Review of Literature**: Purpose of the Review, Identification of related Literature, Organizing the related Literature, Archive.

**Research Report**: General format of Research report, writing technical research report, writing a research proposal, research paper, chapter writing, Ph. D thesis, Erratum, Proof reading, Keywords and Phrases, Mathematical subject Classifications and indexing, Short communication, fast track communication of a research paper, Postal/Oral presentation, Plenary talks, Invited talks of a conference/ workshop.

**Databases and Research Metrics:**
**Databases:** Indexing databases, Citation databases: Web of Science, Scopus, etc.
**Research metrics:** Impact Factor of journal as per Journal Citation Report, SNIP, SJR, IPP, Cite Score.
Metrics: h index, g index, il0 index, altmetrics

**Preliminaries of computer system**: Introduction to computer, Basic concept of computer hardware, software, operating systems, Algorithm and Flowchart, programming languages, representation of numbers in computers, Scientific text processing packages, Curves, Surface drawing packages, Basic concepts of programming with Mathematica and MATLAB.

**Subject Specific Research Methodology**: Groups, rings, fields and their applications. Applications of Linear Operators and system of linear equations. Applications of continuous and differentiable functions in Euclidean spaces. Applications of Weierstrass Approximation Theorem. Applications of continuity, compactness and completeness in metric spaces.
Applications of Cauchy-Goursat theorem, Cauchy integral formula, uniqueness (identity) theorem and Rouche's theorem. Taylors and Laurent series in complex plane. Application of existence and uniqueness theorem for ODE, Applications of Bessel, Legendre, Gauss hypergeometric functions. Application of Integral Transformations and Integral Equations, Lagrangian and Hamiltonian Equation of Motion in Dynamical Systems, Generalized Coordinates.

**GROUP B: MATHEMATICS:**

**Abstract Algebra:** Groups: Review of basic concepts of Group Theory: Lagrange's Theorem, Cyclic Groups, permutation Groups and Groups of Symmetry:$S_n$, $A_n$, $D_n$, Congugacy Classes, Index of a subgroup, Divisible Abelian Groups. Homomorphism of Groups, Normal Subgroup, Quotient Groups, Fundamental Theorem (Structure Theorem) of finite Abelian Groups, Cauchy's Theorem, Group Action, Sylow Theorems and their applications, Solvable Groups (Definitions and Examples only).
Ring: Ideals and Homomorphism, Prime and Maximal Ideals, Quotient Field of an Integral Domain, Polynomial and Power Series Rings. Divisibility Theory: Euclidean Domain, Principal Ideal Domain, Unique Factorization Domain, Gauss Theorem, Eisenstein's criterion.

**Linear Algebra:** Review of Vector Space: Vector Spaces over a field. Subspaces. Sum and direct sum of subspaces. Linear span. Linear dependent and independence. Basis. Finite Dimensional spaces. Existence theorem for bases in the finite dimensional case. Invariance of number of vectors in basis, dimension. Existence of complementary subspace of any subspace of a finite dimensional vector space. Dimensions of subspaces. Quotient space and its dimension.
Matrices and Linear transformations: Matrices and linear transformations, change of basis and similarity. Algebra of linear transformations. The rank-nullity theorem. Change of basis. Isomorphism Theorems. Dual space. Bi-dual space and natural isomorphism. Adjoint of linear transformations. Eigen values and eigenvectors of linear transformation. Determinants. Characteristic and minimal polynomials of linear transformations, Cayley- Hamilton Theorem. Annihilators. Diagonalization of operators. Invariant subspaces and decomposition of operators. Canonical forms.
Inner Product Spaces: Inner product spaces. Cauchy-Schwartz inequality. Orthogonal vectors and orthogonal complements. Orthogonal sets and bases. Bessel's inequality. Gram-Schmidt orthogonalization method. Hermitian, Self Adjoint, Unitary and Orthogonal transformation for complex and real spaces. Bilinear and Quadratic forms, Real quadratic forms.

**Real Analysis:** Bounded Variation: Functions of Bounded Variation and their properties, Riemann Stieltjes integral and its properties, Absolutely Continuous Functions.
The Lebesgue Measure: Lebesgue Measure: (Lebesgue) Outer measure and measure on R, Measurable sets form an σ-algebra, Borel sets, Borel σ-algebra,open sets, closed sets are measurable functions, non-measurable set, Measure space, Measurable Function and its Properties, Borel measurable functions, Concept of Almost Everywhere (a.e.), sets of measure zero, Steinhaus Theorem, Sequence of measurable functions, Egorov's Theorem, Application of Lusin Theorem.
The Lebesgue Integral: Simple and Step Functions, Lebesgue integral of simple and step functions, Lebesgue integral of a bounded function over a set of finite measure, Bounded Convergence Theorem, Lebesgue integral of non-negative function, Fatou's Lemma, Monotone Convergence Theorem. The General Lebesgue integral: Lebesgue Integral of an arbitrary Measurable Function, Lebesgue Integrable function. Dominated Convergence Theorem. Convergence Theorem. Convergence in Measure. Riemann Integral as Lebesgue Integral. Product measure spaces, Fubini's Theorem (applications only).

**Ordinary Differential Equations & Special Functions:** Ordinary Differential Equations: Existence and Uniqueness: First order ODE, Initial Value Problems, Existence theorem, Uniqueness, basic theorems, Ascoli-Arzela Theorem (statement only), Theorem on convergence of solution of initial value problems, Picard-Lindelof theorem (statement only), Peano's existence theorem (statement only) and corollaries.
Higher Order Linear ODE: Higher Order Linear ODE, fundamental solutions, Wronskian, Variation of parameters.
Boundary Value Problems for Second Order Equations: Ordinary Differential Equations of the Strum-Liouville type and their properties, Application to Boundary value problems, Eigenvalues and Eigenfunctions, Orthogonality theorem, Expansion theorem. Green's function for Ordinary Differential Equations, Applications to Boundary Value Problems.

Special Functions: Singularities: Fundamental System of Integrals, Singularity of a Linear Differential Equation. Solution in the neighbourhood of a singularity, Regular Integral, Equation of Fuchsian type, Series solution by Frobenius method.

Hypergeometric Equation: Hypergeometric Functions, Series solution near zero, one and infinity. Integral Formula, Differentiation of Hypergeometric Functions.Legendre Equation: Legendre Functions, Generating Function, Legendre Functions of First & Second kind, Laplace Integral, Orthogonal Properties of Legendre Polynomials, Rodrigue's Formula.

Bessel Equation: Bessel's Functions, Series Solution, generating Function, Integral Representation of Bessel's Function, Hankel Functions, Recurrence Relations, Asymptotic Expansion of Bessel Functions.

**Calculus of Several Variables:** $\mathbb{R}^n$ as a normed linear space, L($\mathbb{R}^n$, $\mathbb{R}^m$ ) as a normed linear space. Limits and continuity of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$. The derivative at a point of a function from $\mathbb{R}^n$ to $\mathbb{R}^m$ as a linear transformation. The tangent space and linear approximation. The chain rule. Partial derivatives and higher order partial derivatives and their continuity. Sufficient conditions for differentiability. Comparison between the differentiability of a function from $\mathbb{R}^2$ to $\mathbb{R}^2$ and from $\mathbb{C}$ to $\mathbb{C}$. Examples of discontinuous and non-differentiable functions whose partial derivatives exist. $\mathbb{C}^1$ maps. Euler's theorem. Sufficient condition for equality of mixed partial derivatives. Proofs of the Inverse Function Theorem, the Implicit Function theorem, and the Rank Theorem. Jacobians. The Hessian and the real quadratic form associated with it. Extrema of real valued functions of several variables. Proof of the necessity of the Lagrange multiplier condition for constrained extrema. Riemann integral of real valued functions on Euclidean spaces, measure zero sets, Fubini's Theorem. Partition of unity, change of variables. Stoke's theorem and Divergence Theorem for integrals.

**Complex Analysis:** Complex numbers: Complex Plane, Stereographic Projection.Complex Differentiation: Derivative of a complex function, Comparison between differentiability in the real and complex senses, Comparison between the real and complex differentiability via ℝ-linear and ℂ-linear maps, Cauchy-Riemann equations, Necessary and sufficient criterion for complex differentiability, Analytic function, Entire function, Harmonic functions and Harmonic conjugates.

Complex Functions and Conformality: Polynomial functions, Rational function, Power series, Exponential, Logarithmic, Trigonometric and Hyperbolic functions, Branch of a logarithm, Conformal maps, Mobius Transformations. Complex integration: The complex integral (over piecewise $C^1$ curves), Cauchy's Theorem and Integral Formula, Power series representation of analytic functions. The difference between real analytical function and $C^\infty$-functions over R. Real Analyticity vs. Complex Analyticity. Morera's Theorem, Goursat's Theorem, Liouvill's Theorem, Fundamental Theorem of Algebra, Zeros of analytical function, Identity Theorem, Weierstrass Convergence Theorem, Maximum Modulus Principle and its applications, Schwartz's Lemma, Index of a closed curves, Contour, Index of contour Simply connected domains, Cauchy's Theorem for simply connected domains. Singularities: Uniform convergence of sequence and series. Laurent series, Casorati-weierstrass Theorem, Poles, Residues Theorem and its application to contour integrals, Meromorphic function, Application of Argument Principle, Application of Rouche's Theorem.

**Topology:** Review of Metric space: Examples and its properties. Topological Space and Continuous functions: Topology on a set, Examples of Topologies (Topological Space): Discrete Topology, Indiscrete Topology, Finite Complement Topology, Countable Complement Topology, Topologies on the real line: $\mathbb{R}_l$, $\mathbb{R}_k$ ,ℝ with usual topology etc., Finer and Coarser Topologies, Basis and sub basis for a Topology, Product Topology on X×Y , Subspace Topology, Interior Points, Limit Points, Derived Set, Boundary of a Set, Closed Sets, Closure and Interior of a Set, Kuratowski closure operator and the generated Topology. Continuous functions, Rules for Constructing Continuous Functions: Inclusion Map, Composition, by restricting the domain, by restricting/expanding the range, Pasting Lemma, Open Maps, Closed Maps and Homeomorphisms, Embedding of a Topological Space into another Topological Space (Example only).

(Infinite) Product Topology: Sub basis for product Topology defined by Projection Maps, Box Topology, Metric Topology.

Connectedness and Compactness: Connected and path Connected Space: Definitions, Examples and its simple properties, Connected subsets of the real line, Introduction to Components and path Components, Local Connectedness, Compact Spaces, Compact subset of the real line, Heine-Borel Theorem.

**Partial Differential Equations:** First Order P.D.E.: Formation and solution of PDE, Integral Surface, Cauchy problem order equation, Orthogonal Surface, First order non-linear PDE, Characteristics, Compatible system, Charpit's method. Classification and canonical forms of PDE.

Second order Linear P.D.E.: Classification, reduction to normal form: Solution of equations with constant coefficients by (i) factorization of operators (ii) separation of variables.

Elliptic Differential Equations: Derivation of Laplace and Poisson equation, Boundary value problem, Separation of variables, Dirichlet's problem and Neumann problem for a rectangle, Interior and Exterior Dirichlet's problem for a circle, Interior Neumann problem for a circle, Solution of Laplace equation in cylindrical and spherical coordinate, Examples.

Parabolic Differential Equations: Formation and solution of Diffusion equation, Dirac Delta function, Separation of variables method, Solution of Diffusion Equation in Cylindrical and Spherical coordinates, Examples.

Hyperbolic Differential Equations: Formation and solution of one-dimensional wave equation, canonical reduction, Initial Value Problem, D'Alembert's solution, Vibrating string, Forced Vibration, Initial Value Problem and Boundary Value Problem for two-dimensional wave equation, Periodic solution of one-dimensional wave equation in cylindrical and spherical coordinate systems, vibration of circular membrane, Uniqueness of the solution for the wave equation, Duhamel's Principle, Examples.

Green's Function: Green's function for Laplace Equation, methods of Images, Eigen function Method, Green's function for the wave and Diffusion equations. Laplace Transform method: Solution of Diffusion and Wave equation by Laplace Transform.

**Functional Analysis:** Banach Spaces: Normed Linear Spaces and its properties, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Riesz Lemma. Bounded Linear Transformations. Uniform Boundedness Theorem, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Necessary and sufficient conditions for Bounded (Continuous) and Unbounded (Discontinuous) Linear functional in terms of their kernel. Hyperplane, Necessaryandsufficient conditions for a subspace to be hyperplane. Applications of Hahn-Banach Theorem, Dual Space, Examples of Reflexive Banach Space. $L^p$ -Spaces and their properties.

Hilbert Spaces: Real Inner Product Spaces and its Complexification, Cauchy-Schwartz Inequality, Parallelogram law, Pythagorean Theorem, Bessel's Inequality, Gram-Schmidt Orthogonalization Process.

Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval's Identity, Orthogonal Complement and Projections. Riesz Representation Theorem for Hilbert Spaces, Adjoint of an Operator on a Hilbert Space with examples, Reflexivity of Hilbert Spaces, Definitions and examples of Self-adjoint Operators, Positive Operators, Projection Operators, Normal Operators and Unitary Operators. Introduction to spectral Properties of Bounded Linear Operators.

**Classical Mechanics:** Hamilton's principle and principle of least action. Hamilton's canonical equations. Canonical transformation with different generatingfunctions. Lagrange and Poisson brackets and their properties. Hamilton-Jacobi equations and separation of variables. Routh's equations, Poisson's identity. Jacobi-Poisson Theorem. Brachistochrone problem. Configuration space and system point. Variation of functional, Necessary and sufficient conditions for extrema, Euler-Lagrange's equations and its Applications: Geodesic, minimum surface of revolution, Brachistochrone problem and other boundary value problems in ordinary and partial differential equations.

Numerical Analysis: Numerical Solution of System of Linear equations: triangular factorization methods, matrix inversion method, Iterative Methods-Jacobi method, Gauss Jacobi method, Gauss Seidel method, Successive over relaxation (SOR) method and convergence condition of Iterative methods, Rate of Convergence of methods.

Solution of Non-linear Equations: Iteration methods: Tchebyshev method, Multipoint method, Modified Newton-Raphson method (for real roots simple or repeated), Rate of convergence of all iteration methods. System of Non-linear Equations: Newton's method, Quasi-Newton's method.

Numerical solution of Initial Value Problem for ODE: First order Equation: Runge-Kutta methods, Multistep predictor-corrector methods, Convergence and stability.

Two Point Boundary Value Problem for ODE: Finite difference method, Shooting Method. Numerical Solution of PDE by Finite Difference Method: Parabolic equation in one dimension (Heat equation), Explicit finite difference method, Implicit Crank-Nicolson method, Convergence and stability.

**Mathematical Methods:** Laplace Transform: Laplace Transform, properties of Laplace transform, inverse formula of Laplace transform (Bromwich formula), Convolution theorem, Application to ordinary and partial differential equations.

Fourier Transform: Properties of Fourier transform, Inversion formula, Convolution, Parseval's relation, Multiple Fourier transform, Bessel's Inequality, Application of transform to Heat, Wave and Laplace equations.

Hankel Transform: Hankel transform, Inversion formula of Hankel transform, Parseval relation, Finite Hankel transform, Application to partial differential equations.

Integral Equation: Basic concepts, Volterra integral equations, Relationship between linear differential Equations and Volterra equations, Resolvent kernel, Method of successive approximations, Convolution type equations, Volterra equations of the first kind, Abel's integral equation, Fredholm integral equations, Fredholm equations of the second kind, the method of Fredholm determinants, Iterated kernels, Integral equations with degenerated kernels, Eigen values and Eigen functions of a Fredholm alternative, Construction of Green's function for Boundary Value Problems, Singular integral equations.