2021

PHYSICS

Paper : PHY - 101

[Mathematical Physics]

(CBCS)

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer any *four* questions. 10×4=40

1. Using Frobenius method find out the solutions of following differential equation $x(x-1)y''+(3x-1)y'+y=0$. 10

2. (a) Prove that : $J_{\frac{3}{2}}(x)\sin x - J_{-\frac{3}{2}}(x)\cos x = \frac{\sqrt{2}}{x^3}\pi$ 4

   (b) Prove that : $\int_0^b J_0(ax)dx = \frac{b}{a}J_1(ab)$ 3

   (c) Prove that : $L_n(0) = n!$ 3

3. (a) Locate the poles and determine their respective orders for the function

   $f(z) = \dfrac{z-\pi}{\sin^2 z}$ 5

(b) Find the residues of the function $f(z) = \cot^2 z$ at the points $z_n = n\pi$ where $n = 0, \pm 1, \pm 2....$ Work out the residue of the function for the case of $n = 0$ using some alternative process. 5

4. (a) Integrate the function; $\dfrac{\exp(1/z^2)}{z^2 + 1}$ along the contour $|z - i| = \dfrac{3}{2}$. 8

(b) Find the value of $\int_{-\infty}^{\infty} e^{-x^2} \left[H_2(x)\right]^2 dx$. 2

5. (a) Prove Plancherel's theorem, by starting with the theory of ordinary Fourier series on a finite interval and allowing that interval to expand to infinity. 5

(b) A function $f(x)$ is defined as —

$$f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & otherwise \end{cases}$$

Show that function can be expressed as —

$$f(x) = \frac{2}{\pi}\int_0^{\infty} \frac{\sin k cos(kx)}{k} dk$$ 5
